

The tools of a planet researcher

Planet researchers need *tools* :

Planet research requires working in shifts, and the researchers come from varied backgrounds. They have to define which units they will use to quantify what is measured. A common language is necessary : these are the mathematical notations.

The mechanisms studied in the Solar System are useful to know, because the same rules also apply to exoplanets, planet mechanics, the Roche limit, and resonances.

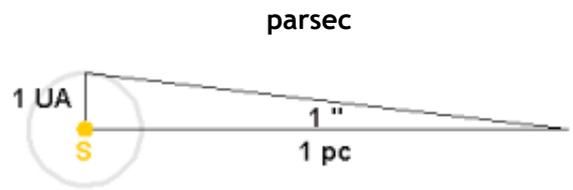
To venture outside the Solar System, we have to know the "laws of the environment", the environment being the Universe : the forces at stake, and the states of the matter. How does a planet/star form? Why does it shine as a black body? How can its light be analysed to detect the signatures of atoms and molecules?

Finally, we use tricks in research because the extrasolar planets are hidden by their larger and brighter stars, the barycenter, and the Doppler effect.

The units used to measure distances

The distance between the Earth and the Sun could have a value of 1 (0.000016, or 1.5×10^{11}). To communicate, planet researchers must first say which UNITS they use!

The International System (S.I. or MKSA) defines a consistent system of units, but it is not always suitable for Astrophysics, because astronomers work with either very small or very immense scales.



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For distances, the unit of the International System (SI) is the *meter*.

- In the Solar System, astronomers use the *astronomical unit*, which is the mean distance between the Earth and the Sun.
1 astronomical unit (AU) = 150 000 000 km = 1.5×10^8 km (more exactly 1.496×10^{11} meters)
- Concerning the stars, astronomers use the *parsec*, which is a shortcut for "per second": a parsec is the distance at which one AU underlies an arc of 1 second. 1 parsec (pc) = 3.26 l.y. = 3×10^{13} km (3×10^{16} m) = 200 000 AU
- The distances between stars or galaxies are given in *light-years*, the distance covered by light in one year.
The speed of light is $c = 300\,000$ km/second.
One light-year (km) = 365(days) * 24(hours) * 3600 (seconds) * 300000(km/second) 1 l.y. = 9.46×10^{15} m = 60 000 AU
!! Warning!!, on these large scales, time plays a significant role : the radiation coming from a star located 2000 light-years away is left over from a star 2000 years ago. It is impossible to know what happened to the star over the past 2000 years!!

The units used to measure masses

The mass unit of the International System is the *kilogram*.

For planets (extrasolar or not), masses are given in terrestrial masses, Jupiter masses, or solar masses

$$M(\text{Terrestrial}) = 6 \times 10^{24} \text{ kg}$$

$$M(\text{Jupiter}) = 2 \times 10^{27} \text{ kg}$$

$$M(\text{Solar}) = 2 \times 10^{30} \text{ kg}$$

!!Important!! Mass and weight are different things, even if, when we are on the surface of the Earth, it is difficult to tell the difference! Weight is equal to mass multiplied by "g", a value almost constant on the surface of the Earth. Weight is related to the gravity force. Mass measures the amount of matter, which depends on the number of atoms and on the mass of these atoms.

On Earth, your weight is the attraction between your mass and the mass of the Earth (see [gravitation](#)) at a distance equal to the radius of the Earth. In the interplanetary medium, you wouldn't have any weight (you would be in a state of weightlessness), but you would keep the same mass. On the surface of the moon, your weight would be 6 times lower than your weight on Earth.

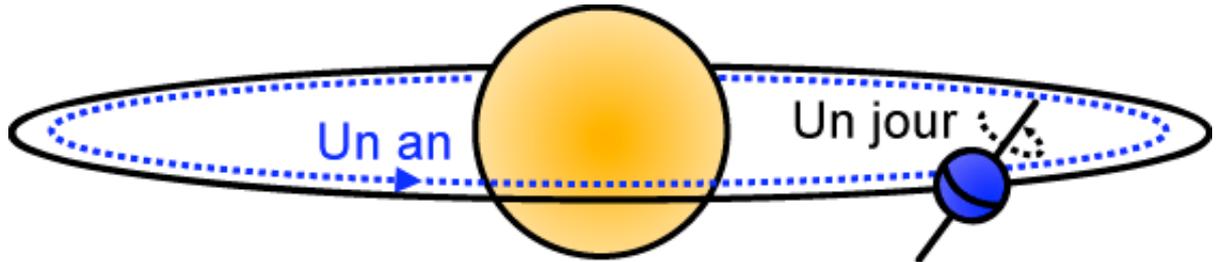
Des poids et des masses



The units used to measure time

The time unit of the International System is the *second*.

The main units of time in astronomy are the *second* (defined by the vibrations of the Cesium atom) and the *year* (defined by the Sun). Time is given in Universal Time (UT), which is the mean time on Earth, at zero longitude.



The Earth's rotation around its axis defines the day and the revolution of the Earth around the Sun defines the year.
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The units used to measure the brightness of the stars

The unit used to measure the brightness of the stars is *magnitude*: Hipparque, in 200 BC, classified visible stars between the magnitude 0, for the brightest star of the sky (Vega), and 6 for the fainter stars visible to the naked eye.

To keep this scale, we use the formula $m = -2.5 \log\left(\frac{L}{L_0}\right)$ where L_0 is the flux of a star with a magnitude of zero.



The central star has a magnitude of 4,5, the second one has a magnitude of 7,2, and the stars in the sky background have magnitudes of between 15 and 18.

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To check an equation

The first trick to checking an equation is to use units :

To derive the time necessary for light to come from Neptune (4×10^9 km), at a speed of 300000 km/second, is it :

$$time = \frac{distance}{speed} \text{ or } time = distance \times speed ??$$

All you need is to replace this with units :

$$second = \frac{kilometres}{kilometres} \text{ or } second = kilometres \times \frac{kilometres}{second}$$

to see if the first equation is the correct one.

$$time = \frac{4 \times 10^9}{300000} = 13333 \text{ seconds, we divide that by 3600 (seconds/hour) to get 3,7 hours.}$$

Notations

- *decimal numbers* : 2.5 is a better notation for 2,5
- *powers (or exponents)* are useful to give huge quantities (the solar mass in grams for instance) or very small quantities (the mass of a hydrogen atom in grams).

The positive exponent gives the number of zeros :

$$10^3 = 1000$$

The negative exponent also gives the number of zeros, but with a subtraction sign :

$$10^{-3} = 0.001 = \frac{1}{1000}$$

The Sun's mass in grams is written : 2×10^{33} , which is also written with 2 followed by 33 zeros.

The mass of a hydrogen atom in grams is written : 1.7×10^{-25} , which is also written with 25 zeros followed by 17, with a dot after the first zero.

- *logarithms* : this is the function that gives the power of a number :
The decimal logarithm or "log" of 10^{33} is 33

This replaces some multiplications with some additions (it is easier to do). It is more difficult to calculate $0.0001 \times 10000000000$ than it is to calculate $10^{-4} \times 10^{10}$: it is sufficient to add exponents to find 10^6 , which is one million.

Logarithms are very useful for the planet researcher who juggles billions of stars and the thousandths of arcseconds. $\log(xy) = \log(x) + \log(y)$

A function

Volume d'une sphère en fonction du rayon



A *function* shows how one quantity varies compared to another one.

For instance, the function

$$\text{volume}(\text{radius}) = \frac{4\pi \text{radius}^3}{3}$$

shows how the *volume of a sphere* varies from the *radius*.

$\pi = 3.1415926\dots$ is the *pi* constant.

!Important!

- Radius and volume units are related.
- When the radius is multiplied by 2, the volume is multiplied by 8.

The mechanics of planets

It is possible to predict the position of planets in the future or in the past with a precision of within a few centimeters, or better. Three people, Tycho Brahé, Johan Kepler and Isaac Newton have left their marks on what is now called "celestial mechanics", which is the science of computing the movements of the objects of the Solar System.

Tycho Brahé carried out observations on the position of the planets in the sky with an unequalled precision at that time (1600). These very accurate observations, over a long period of time, enabled Johan Kepler to show that the planets were moving on elliptical orbits, and that the planets were moving faster when they were closer to the star. The equations that define this movement are the three Kepler laws.

QUADRANS MURALIS SIVE TICHONICUS



Tycho Brahé in Uraniborg Observatory : he uses very accurate visualization instruments.

Copyright : Library of Paris Observatory.

Kepler's laws

- Kepler's first law
Orbits are ellipses within which the Sun is one of the focuses (animation 1 of an ellipse).
- Kepler's second law
The "area law" : the velocity along the orbit is such that the area swept from the Sun is constant. When the planet is closer to the Sun, it goes faster. (animation 2 of an ellipse).
- Kepler's third law
The period P of revolution around the Sun increases with the semimajor axis*, D, and

$$\frac{P^2}{D^3}$$

is constant for all the planets of the Solar System.

* : the semimajor axis is the mean distance from the Sun.

If P is in terrestrial years and D in astronomical units, the constant is equal to 1.

For a planet at 5 AU,

$$P^2 = 5^3 = 125$$

$$\text{then } P = \sqrt{125} = 11.2 \text{ years}$$

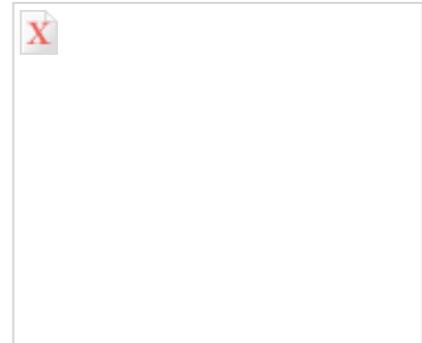


Illustration of Kepler's second law.

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The law of gravity

Newton, at the end of the 17th century, understood that the movement of the planets, as well as the movement of the moon, is due to the gravitational force and that this same force is also the cause of "weight" on Earth (the apple).

Two objects of masses M_1 and M_2 attract each other with a force proportional to the masses and in inverse proportion to the square of the distance between them.

$$F = \frac{GM_1 M_2}{D^2}$$

G is a constant equal to $6,67259 \frac{m^3}{kg s^2}$

The Kepler equations are a consequence of this expression.

When we consider more than two bodies, each object is submitted to the forces of the other bodies, and it is not possible anymore to directly find the movement of these objects. Even the 3-body problem is very complicated. Thus, the theory of the movement of the moon when the Sun and the Earth are present is an entire life's work - that of Charles-Eugène Delaunay, whose work is still useful now for computer simulations.

The Roche limit

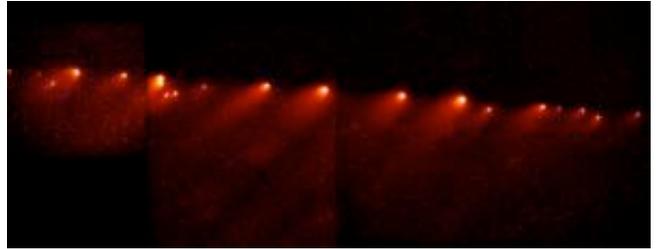
The Roche limit is the minimal distance, with respect to the center of a planet, at which a satellite is able to orbit without being destroyed by tidal forces. If the planet and the satellite have the same density, the Roche limit is 2.5 times the radius of the planet. Within this limit, the satellite is destroyed by tidal forces.

Each of the ring systems in the Solar System are within the Roche zone of their planets.

Solid satellites can exist inside the Roche zone if they are sufficiently small, since the tension of the rocks prevent them from breaking up.

In a disk of remnants around a newly formed planet, the matter outside the Roche limit can form satellites, whereas nearer to the planet, the tidal forces prevent the formation of any satellite.

This mechanism still holds in the neighbourhood of a star : no planets can exist nearer than 2.5 times the radius of its star.



The Shoemaker Levy 9 comet fragmented by Jupiter in 1994.

Copyright : NASA/HST

Calculation of the Roche limit

Roche's argument is as follows : although the satellite is spherical, let us suppose that it is made up of two spheres of radius r and mass m . Let us think about two snow balls of radius r , held together by the gravitation force that each sphere exerts on the other. That force, F_{att} , is given by Newton's relation :

$$F_{att} = \frac{Gm^2}{(2r)^2}$$

Now, let us consider that each satellite is at a distance D of a planet of mass M and radius R . The attraction force F , between the planet and the nearest snow ball, will be greater than the force F' between the planet and the more distant ball. The force F is given by the following relation :

$$F = \frac{GMm}{D^2}$$

And force F' is given by :

$$F' = \frac{GMm}{(D+2r)^2}$$

The two balls will feel the resulting force F_{mar} , which tends to separate them. This force is equivalent to the difference between the forces F and F' . Therefore we have : $F_{mar} = F - F'$ And, because $D \gg r$:

$$F_{mar} = -\frac{4GMmr}{D^3}$$

The two masses will be separated if the force F_{mar} is greater than the force F_{att} , that is if :

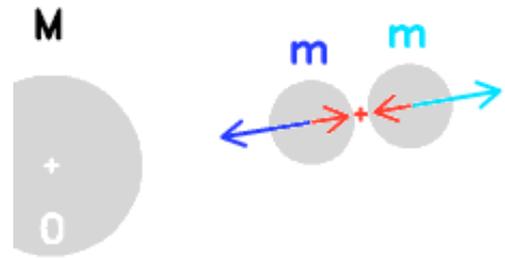
$$\frac{2^4 M}{D^3} > \frac{m}{r^3}$$

Let us now replace the mass M by $\rho_P \frac{4}{3} \pi R^3$, where ρ_P is the density of the planet, and the mass m by

$$\rho \frac{4}{3} \pi r^3, \text{ where } \rho \text{ is the density of the satellite.}$$

Therefore, the two masses are separated if the distance D is less than $2^{\frac{4}{3}} R \left(\frac{\rho_P}{\rho} \right)^{\frac{1}{3}}$

. This is a quite satisfactory approximation since $2^{\frac{4}{3}}$ is equal to 2.51, whereas the exact value is 2,456.



Equilibrium or rupture, under the effect of the self-gravitational field, which must ensure cohesion, and of the gradient terms of the gravitational field, which tears the satellite apart (in the frame of reference of the center of mass of the satellite).

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What is a resonance?

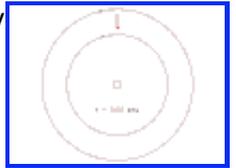
An example of *resonance* is the action made to launch a *swing*:

To move a swing, we must give it an impulse at each passage, or maybe every two or three passages. In order to maintain the motion, it is important to give the impulses with the swing in the same position : the impulses must be in *resonance* with the swing.

The impulse *period* (the time between two impulses) must be equal to the *period* of the swing (swinging duration), or equal to twice (or triple) the period of the swing.

Around a star, the bodies do not move at the same velocity and they gravitationally interact.

- If two planets are not in resonance, their successive perturbations get muddled up and yield small displacements of the planets.
- If the planets are in resonance, their successive perturbations are added to each other and become important.
- However, a resonant configuration between two bodies can also correspond to an equilibrium position where the two objects "get in each other's way" the least.



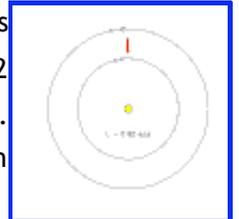
Where are the resonances?

In a system of bodies orbiting a central primary (planets around a star, satellites around a planet), possible resonances exist between these moving bodies.

They occur when there is a commensurable ratio between orbital parameters.

The most "simple" resonances concern revolution periods. They are called "mean motion resonances". The "mean motion" is the angular speed n_N , with n_P the revolution period.

There is a resonance if the mean motion of the planet $2n_N = 3n_P$, $2P_1 = P_2$, is commensurable with that of the planet $n_1 = 2n_2$, n_1 . For instance, there is a 1/2 resonance if the planet n_2 is twice as fast as planet 1, which can be written 2, or 1. There is a resonance of order n/m, where n and m are two integers, if a planet makes n revolutions when the other one makes m revolutions.



In Neptune's rings, the edge of the Adams ring is in a resonance 42:43 with the satellite Galatea.

Resonances between the rotation motion of a body and its revolution motion are also possible.

When the orbits are eccentric and/or inclined, other "velocities" can appear. The mutual perturbations between planets make their orbits : this is [precession](#). The precession velocities can also be in resonance between themselves and/or with the mean motions.

[Gravitation](#) is a very simple law that leads to complex phenomena.

Resonances are everywhere in the Solar System :

- The rotation motion of the moon on itself is in a 1:1 resonance with its revolution motion. The same time is necessary to rotate or to make a revolution around the Earth. That is the reason why, from Earth, we always see the same side of the moon.
- The rotation motion of Mercury on itself is in a 3:2 resonance with its revolution motion around the Sun. The planet rotates three times when it makes two revolutions around the Sun.
- Neptune and Pluto are in a 3:2 resonance. Neptune makes three revolutions around the Sun when Pluto makes two. The angular velocity of Neptune, 2 , is greater than that of Pluto, $n = \frac{2\pi}{P}$. It can be written P
- In Saturn's rings, there are numerous resonances between satellites, or between satellites and rings.

What parts do resonances play?

There are various consequences of resonance :

- In the asteroid belt, it is well known that there are gaps, i.e. some zones where no objects are present. *These locations are in resonance with Jupiter*. The role of Jupiter has been to *eject* the bodies in resonance with the planet.
- The 3:2 resonance with Neptune plays the inverse role. At the location of this resonance, numerous bodies piled up, including Pluto. This resonance is *stable*.
- The resonance between the revolution period and the rotation period of the Moon *stabilizes* its synchronous rotation with the Earth. This configuration saves the Moon internal frictions that it was undergoing when rotating faster, because of the tidal effects of the Earth. These frictions slowed the Moon down to this equilibrium position. This is a very efficient mechanism. In the Solar System, all of the nearby satellites are in synchronous rotation.
- In Saturn's rings, a huge number of *structures: sharp edges, wakes, density waves*, are created by resonances with satellites.

The forces in the Universe

The interactions of matter in the Universe are explained by four force types :

The electromagnetic force concerns electrically charged particles. Therefore, it is attractive between two particles of opposite signs, and repulsive between two particles of the same sign. It varies as $\frac{1}{r^2}$. It is responsible for the cohesion between atoms : the protons of the nucleus and the electrons attract each other. It is not perceptible on large scales because there are as many protons as electrons in the Universe, the matter is "neutral" on large scales.

The strong force of interaction : this is very intense and acts on very small scales (10^{-15} meters). It concerns the protons and the neutrons. It enables the nuclei to remain "stuck" together, despite the repulsive effect of the electromagnetic force on the protons.

The weak force of interaction acts on all the particles on very small scales (10^{-18} meters), it is responsible for beta radioactivity.

The gravitational force concerns *all* masses, and therefore all particles. It can be understood as a geometric property of space-time matter. It varies as $\frac{1}{r^2}$ until infinity. This force is responsible for the large scale structure of the Universe.

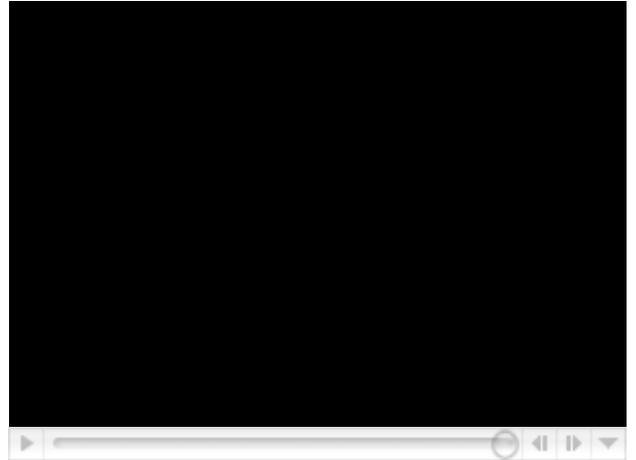
Physicists are searching to unify these forces in a comprehensive theory which could give an account of both general relativity and quantum mechanics. No complete theory exists now. However, weak interaction and electromagnetism are united in a single theory : the electroweak theory.

The states of matter

Once made by stars, atoms are almost eternal. On the other hand, they can combine to form molecules, which are organized differently, depending on the temperature and pressure of the medium. Thus, oxygen and hydrogen atoms are a billion years old. On the other hand, water molecules (H_2O) can easily break up to give back hydrogen and oxygen.

We know three principal *states* of matter :

- The solid state : atoms (or molecules) are very tight.
- The liquid state : atoms (or molecules) can move around each other.
- The gaseous state : atoms (or molecules) are distant from each other.



Water molecule.

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The atoms can also lose electrons and become ions. When the matter is made up of separate nuclei and electrons, we speak of plasma.

Heating a body shakes up atoms/molecules, and makes matter pass from the solid state to the liquid and then gaseous ones. The temperature, which makes the matter pass from one state to another, depends on the matter and on the pressure conditions. The same amount of matter (i.e. the same number of atoms/molecules) takes up more space in the gaseous state than in the liquid or solid state.

The states of matter in the Universe

The interstellar medium (ISM)

The interstellar medium is made up of gas, between 10 and 100K. The chemistry there is slow.

The stars

The stars are made up of dense ionized gas. The temperature on the surface of the stars is between 3000 and 50000K. At this temperature, the atoms are separated into plasma (ion soup).

The giant planets

The giant planets are made up of gas with a liquid and/or solid core. The giant planets of the Solar System are between 100 and 200K.

The terrestrial planets

The terrestrial planets are made up of solid materials and an atmosphere. Nearer to the Sun than the giant planets, their temperatures are between 200 and 500K. The combined effect of the effective temperature and the greenhouse effect maintains a temperature interval which enables water to be in the liquid state (0-100 degrees Celsius or 273-373K), a favorable medium for the emergence of life.

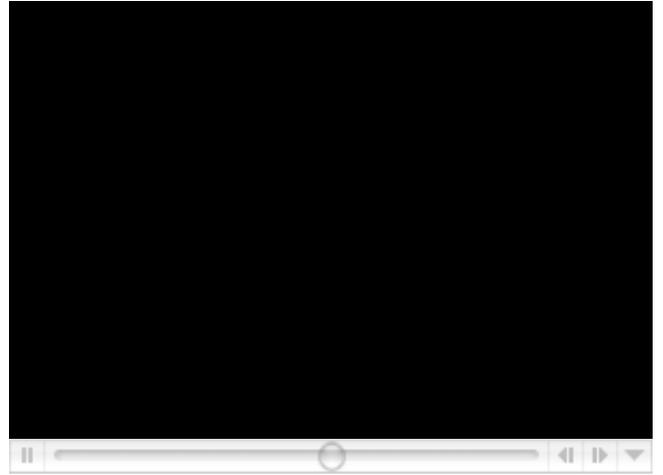
The main difference between terrestrial and giant planets concerns their density : one cubic centimeter of a terrestrial planet weights between 4 and 6 grams whereas one cubic centimeter of a giant planet weighs between 1 and 2 grams.

A gas cloud collapses on itself

The story begins in interstellar space : a wave propagates in the gas (probably due to the explosion of a nearby supernova) and part of the cloud starts to shrink.

The cloud collapses, the density and temperature increase in the center. Becoming smaller, the cloud starts to rotate and takes the shape of a disk around a central mass.

What happens next depends on the amount of matter in the central mass...



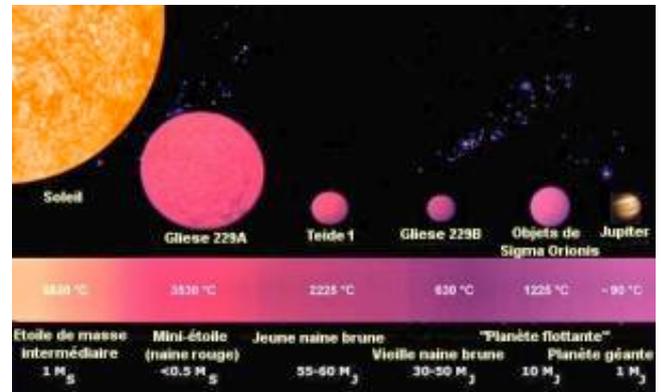
The formation of a planetary system.

Copyright : Paris Observatory / UFE

A choice between a star, a brown dwarf or a planet

- If the mass of the central core is small, , $M < 3 \times 10^{28}$ kg i.e. $13M_J$ (Jupiter's mass), it becomes a gas sphere and forms a giant planet (see [what is an exoplanet ?](#)).
- If $13M_J < M < 100M_J$ (10 percent of the mass of the Sun), a brown dwarf forms, where the nuclear combustion only concerns the fusion of the deuterium (D) into helium (He).
- If $M > 0.1M_{\odot}$ (solar mass), the central temperature becomes greater than 5×10^6 K and the fusion of hydrogen (H) into helium (He) starts, lighting up a star.

Between star and planet



Temperature and diameters of several brown dwarves, compared to the Sun and Jupiter.

Copyright : ESA / Medialab. Data from R. Rebolo and Serge Jodra

These three scenarios can occur in the neighbourhood of a forming star. This leads to a system of double stars (which are numerous in the Universe), or to a system associating a star with a gas giant or a star with a brown dwarf.

If the scenario above concerns an isolated gas condensation, an isolated star forms. If the mass is small, an isolated brown dwarf, or even an isolated planet, can form.

Some isolated brown dwarves have been detected. However, it is difficult to detect them since they are faint, and shine for a short time in the infrared.

The formation of isolated planets (called floating planets) is theoretically possible. However, nothing has been detected yet. These objects do not have any energy sources, and do not emit any radiation.

Now, we consider a case where the body formed is a star, and look at what happens in its environment. Because that's where the "real" planets form in a circumstellar disk, as the Earth did.

In the environment of a star

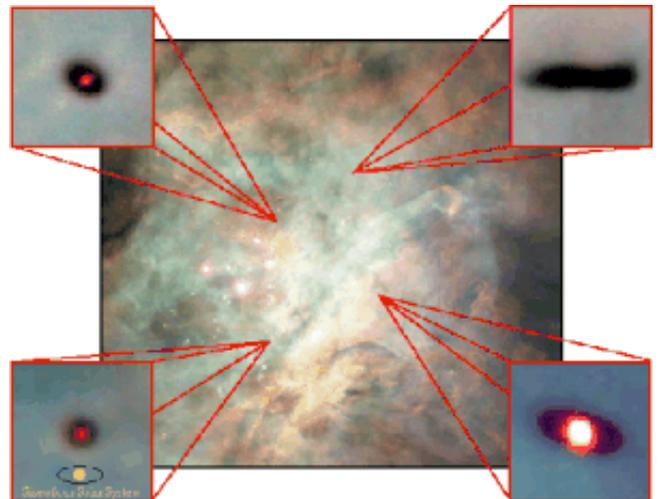
When a star "switches on", the collapse of the cloud forms a disk around the proto-star. The study of the Solar System has enabled us to reconstruct the steps which led to the formation of the planets.

- The material forms a very thin dust disk embedded in a thicker gas disk.
- In the dust disk, the grains stick together to form larger bodies, the planetesimals (which look like current asteroids or comets)
- The planetesimals collide to form more massive bodies. As soon as a sufficiently large core forms, it attracts the gas and the planetesimals, it grows faster and forms a planet
- The planets eject the remaining small planetesimals which fall into the Sun, the planets and the satellites (thus forming the numerous craters visible on the Moon or Mercury), or are destroyed in the neighbourhood of the massive planets to form rings. Some of them are sent on the outer edges of the Solar System, forming the Oort cloud, which is a reservoir for current comets.
- The interactions between the planets still create collisions, which can explain the formation of the Moon or the swing of the orbit of Uranus. They can also repel each other and migrate until they find a stable configuration. The [resonance](#) phenomenon plays a large part in the modelling of the orbits of the different bodies of the Solar System.

The dust disk is made up of rocks and metals close from the star, where it is warm. Beyond a certain distance (called the "ice limit"), the lower temperature enables the ice to form. Because hydrogen (H) and oxygen (O) are the most abundant [atoms](#) in the Solar System, there are much more planetesimals and the planets form more rapidly. Jupiter, the largest planet, formed at the ice limit, and the other giant planets, made up of gas and ice, are outside this limit.

This scenario of the formation of planets explains quite well all the properties of the planets of the Solar System. But is it still viable for the exoplanets?...

Planetary systems forming in the Orion nebula



The dark regions are dust disks around young stars, where planets are likely to form.

Copyright : NASA / HST / C. R. O'Dell and S. K. Wong

Life and Death of a star

The life of a star starts by a turbulent stage, called T-Tauri, which lasts about one million years. During this stage, the star throws radiations and particles which strongly disrupt the circumstellar disk.

Then, the star is in the « main sequence », the major stage of its evolution (9 billion years for a star like the Sun). When the hydrogen is exhausted in the core, the contraction resumes and the temperature in the center of the star increases. The following of the scenario depends on the mass of the star :

- For the low mass stars ($M < 0.3 M_{\odot}$), the contraction ceases and the star "turns off".
- For the more massive stars ($M > 0.3 M_{\odot}$), the majority of the stars, the temperature in the center reaches 10^8 K, and helium fusion starts.

In the shell, the increase of temperature enables the return of hydrogen fusion into helium. The luminosity, and thus the radiation pressure and the radius of the star, increase dramatically. At the same time, the shell enlarges, gets colder and the center becomes denser. The star becomes a red giant. In the center, the combustion of helium into carbon and oxygen lasts a short time. Following the exhaustion of helium, the mass of the star still determines its evolution.

If the mass is less than $1.4 M_{\odot}$, the diluted envelope is blown to form a "planetary nebula". The core slowly "turns off", forms a small (R~3000 km) and dense ($\sim 10^{10}$ kg/m³) white dwarf, which cools down very slowly.

For the stars with a mass greater than 1.4 solar masses, the contraction continues. The fusion of the carbon (C), oxygen (O), silicium (Si), manganese (Mg), neon (Ne), and iron (Fe), is fast and releases little energy. After the iron (the most stable element in thermonuclear reactions), there is no available fuel anymore. The contraction resumes. In the center, the temperature increases, which leads to the fusion of elements heavier than iron. However, these reactions consume a lot of energy, which accelerates the contraction. Then, the fusion of the electrons and protons into neutrons occurs. The stellar core collapses in free-fall, to reach a radius of about 10 km and a density of 10^{17} kg/m³. An interior bounce and a shock wave creates a supernova. After the explosion, a very dense central object remains, either a neutron star, or a black hole.

Nébuleuse du Crabe



Ejected matter from a supernova. This event occurred in 1054 and was observed by the Chinese : superimposing of an image in X rays (blue) and in the visible (red). The size of the ring is about one light-year.

Copyright : X rays : NASA/CXC/ASU/J. Hester et al. ; visible : NASA/HST/ASU/J. Hester et al.

The light of the Sun

Electromagnetic radiation is the main source of information in Astronomy.

What we call "light", in common life, is only a very small part of the radiation from the Sun. That's the part the human eye detects.



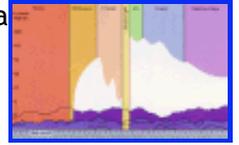
Raindrops scatter the light from the Sun like a prism.

Copyright : Paris Observatory / UFE

The speed of light

The Sun does not emit only visible light. It also emits radio, infrared, UV, X and gamma waves.

- Electromagnetic radiation propagates with a velocity $c = 299790 \text{ km/s}$.
- 8 minutes and 22 seconds are necessary for light to travel from the surface of the Sun to the Earth.



Parts of the radiation are absorbed by the atmosphere and do not reach the ground. To observe the sky in these wavelengths, we must send receivers into space.

The quantities which define this light are :

- The frequency, f , is the number of pulses per second (hertz is the frequency unit)
- The period, $P = \frac{2\pi}{f}$, is the length of one pulse (in seconds)
- The wavelength $\lambda = c \times P$ (the unit is the meter)
- The energy $E = h \times f$ (the unit is the Joule)

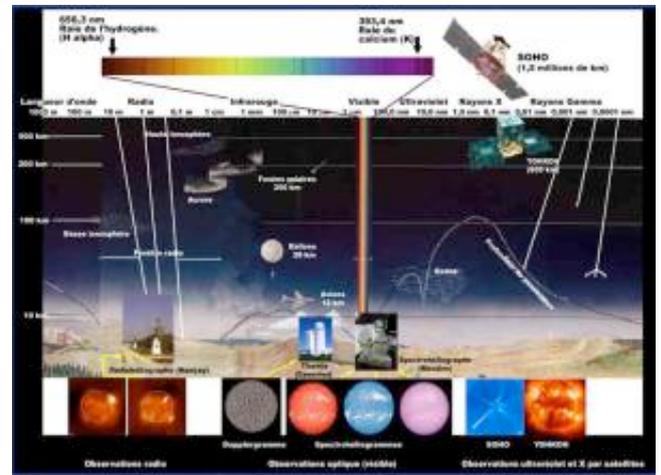


h is the Planck constant : $h = 6.626 \times 10^{-34} \text{ joule.seconde}$

The electromagnetic spectrum

These radiations are all of the same nature. The different regions of the electromagnetic spectrum have different names because they correspond to different detectors and different emission mechanisms.

The different sources of radiation



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The black body

A star, or a planet, emits electromagnetic radiation made up of a continuous spectrum (black body) which depends on its temperature, emission, and absorption rays which depend on the matter located between the object and the telescope.

A *black body* strongly interacts with the radiation that it emits, it is "opaque". It absorbs all the energy it receives, and emits a radiation that depends on its *temperature*, in all wavelengths. The warmer it is, the more its light is shifted to the short wavelengths.

The Wien law gives the wavelength of the emission

maximum : λ_{max} (in meters) = $\frac{0.003}{T(K)}$. It enables us

to define a relationship between temperature and color, via the correspondence between wavelength and color.

Thus, we have a thermometer : a blue star is warmer than a red one.

For instance, the human body is at 37° Celsius, i.e. 310 K. $\lambda_{max} = 9.7 \times 10^{-6}$ meters. The human body emits radiation in the infrared.

The Sun has a temperature of 5780 K. Its emission is strong in the visible wavelengths. Probably the human eye adapted to "see" the region of the electromagnetic spectrum where the radiation of the Sun is the strongest.

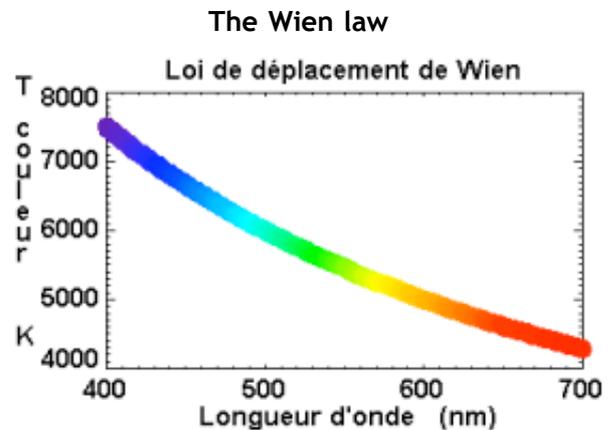
The temperature of a body T corresponds to a thermal disturbance velocity given by : $\frac{1}{2} m V^2 = \frac{3}{2} k T$ where m is the mass and k the Boltzmann constant.

$$k = 1.38 \times 10^{-23} \frac{\text{joule}}{\text{Kelvin}}$$

That's the reason why, if a planet is too warm, the molecules of its atmosphere have a sufficient velocity to escape from the planet. For instance, that's the reason why, in the atmosphere of the Earth, there is oxygen but no helium.

The Stefan-Boltzmann law gives the total emitted flux : $F = \sigma T^4$

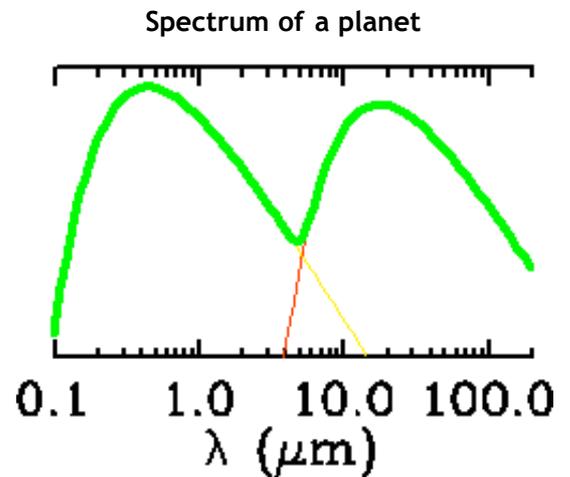
$$\text{Stefan constant : } \sigma = 5.67 \times 10^{-8} \frac{\text{Watt}}{\text{mètre}^2 \text{ Kelvin}^4}$$



The spectrum of a planet

The spectrum of a planet has two bumps :

- One bump is the reflected light of the star, that's the visible part for which the maximum is given by the temperature of the star.
- The second bump is the proper emission of the planet. This component is in the infrared, because the planet is "cold" (a few hundred Kelvins).



The chemical elements

The Universe is mainly made up of *hydrogen* and *helium* atoms. The other elements represent less than 1% of the matter. And yet these rare elements are necessary to form solid matter, ices and rocks, which are the constituents of the terrestrial planets.

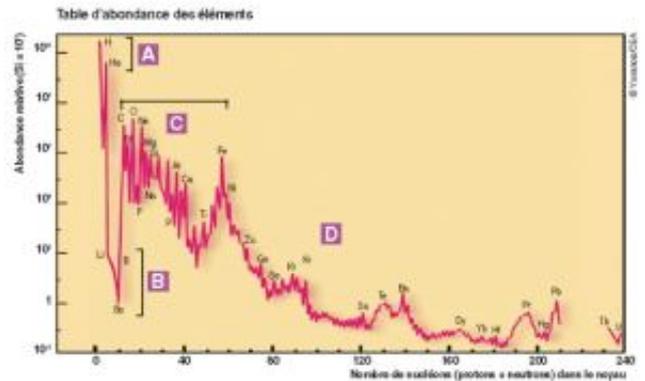
The relative proportion of the elements is the reflection of the processes that have formed these atoms.

On this figure, note that the scale is **logarithmic**, there are 3 intervals between H (hydrogen) and O (oxygen), i.e. there are $10^3=1000$ times as much H than O.

The lightest atoms, hydrogen (H), helium (He), and a little bit of lithium (Li) and beryllium (Be), were created during the Big Bang. Within the stars, the first thermonuclear reactions formed carbon (C), nitrogen (N), oxygen (O) and fluorine (F).

The life of the stars is made up of several steps, and the creation of each atom is associated with one of these steps.

Abundance of the chemical elements in the Solar System



The first planets

Some stars are as old as the the Universe (15 billion years), others have much shorter lifetimes (3 million years).

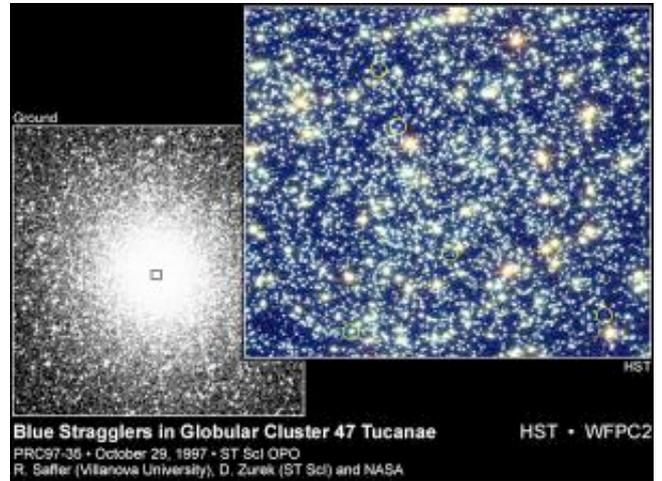
The first-generation stars were only made up of hydrogen and helium. These stars couldn't host solid planets!

They used their H and He to form C, N, O and F atoms. At the end of their life, they became **novae** or **supernovae**, and ejected their matter in the interstellar medium (ISM).

The second-generation stars, formed in this interstellar medium, were made up of H, He, and a little bit of carbon (C), oxygen (O), nitrogen (N) and fluorine (F). The planets of these stars were made up of gas (H and He) and ices : water ice (H₂O), CO ice, and CO₂ ice . The heavy atoms formed by these stars are dispersed in the ISM. Only the stars of the next generations can have rocky and metal planets.

The Sun is 4.5 billion years old. It formed in a medium enriched by several generations of stars.

The **Hubble Space Telescope** observed a lack of planets around very old and metal-deficient (atoms heavier than He) stars, in the Toucan cluster.



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The hydrogen atom

The "classical" model of the hydrogen atom is one electron orbiting one proton.

The electrons are located at specific distances from the nucleus. At rest ($n=1$), the orbit of the electron has a radius of 10^{-11} meters (Bohr classical radius). The electron can also be on larger orbits, associated with integers $n = 2, 3, 4 \dots n = \infty$

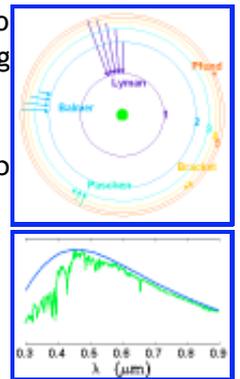
The nucleus has a radius of 10^{-15} meters. The atom at rest is 10000 times bigger than the nucleus. If the nucleus was a coin of 10 centimes, the atom would be a sports ground.

An atom can pass from a fundamental state ($n=1$) to an excited one, absorbing a photon. It can also come back to the fundamental state emitting light, the colour of which (wavelength) depends on the energy levels of the atom.

The transition from a level n_2 to a level n_1 corresponds to an emission/absorption of wavelength λ , such that $\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$, with $R = 1.1 \times 10^7 \text{ m}^{-1}$

If the atom gets a sufficient amount of energy, the electron passes from the level $n=1$ to the level infinity. The atom loses its electron and becomes an *ion*. The corresponding wavelength is 0.91×10^{-6} meters, i.e. in the UV range.

In the atmosphere of a star, the hydrogen atoms, enlightened by the star, only absorb the colours that make them pass from one level to another.

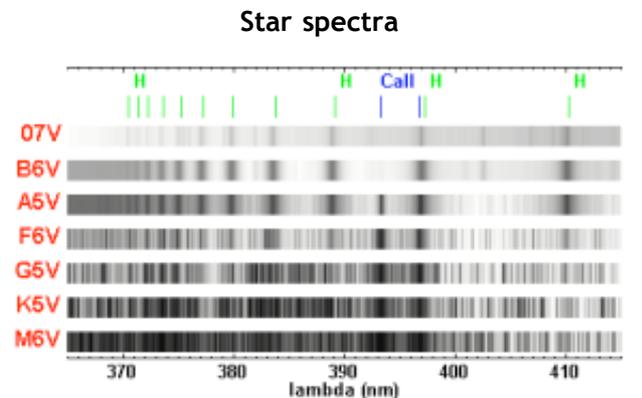


Atoms, molecules and spectra

A molecule is an assembly of atoms. A water molecule is made up of one oxygen and two hydrogen atoms.

Each atom and each molecule is associated with a group of lengths corresponding to energy transitions. The spectrum of a body is the ID of the chemical elements which make it up. The analysis of a complex spectrum gives the chemical composition and the abundance of each element.

Conclusion :The spectrum of a star/planet is therefore made up of a spectrum of a *black body* which depends on its *temperature* , and *absorption rays* which depend on the *composition of its atmosphere*. We see below the spectrum of several stars, from the warmest one (star O) to the coolest one (star M).



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The center of mass



When two objects gravitationally attract each other, they move around each other, around a fixed point, their barycenter or center of mass.

When the two masses are equal, the center of mass is in the middle of the two bodies.

When the masses are different, the barycenter is closer to the most massive body. The ratio of the distances from the center of mass is equal to the mass ratio.

A star and a planet move around their barycenter. Because the planet is much less massive than the star, the barycenter is very close to the center of the star.

In the Solar System, Jupiter is 1000 times less massive than the Sun. The distance between Jupiter and the Sun is 750 million kilometers. The center of mass is located at 750000 kilometers from the Sun, not far from the surface. The other planets, much less massive than Jupiter, alter these movements only slightly.

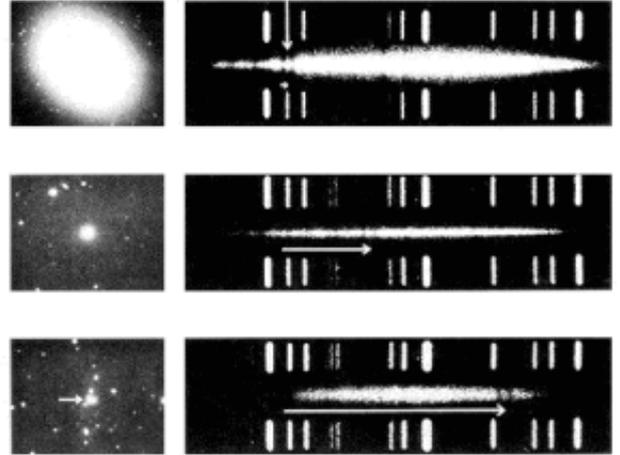
We can use this effect to search exoplanets : the presence of an invisible planet is revealed by the displacement of the star around which it orbits.

The Doppler effect

When a wave is emitted, the sound heard by the listener differs if the transmitter is stationary or moving with respect to the listener.

If its wavelength is λ (meters), the wave emitted has peaks of length λ and period P (seconds), such that $\lambda = cP$, where c is the speed of light (3×10^8 meters per second).

- If the source is stationary, the listener receives a wave whose peaks are separated by a time P , and which the wavelength is λ , equal to the emitted wavelength.
- If the source is moving with a velocity v_r with respect to the observer, the peaks are emitted at intervals P . During the time interval separating the emission of 2 peaks, the source has moved, with a distance $d = v_r P$.



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- If the source moves away from the listener, the second peak covers a greater distance to reach the observer, a distance equal to $\lambda + d$. The listener receives a wave of wavelength

$$\lambda' = \lambda + d = Pc + v_r P = Pc \left(1 + \frac{v_r}{c} \right). \text{ The received wavelength is } \lambda \left(1 + \frac{v_r}{c} \right). \text{ Therefore, it is greater}$$

than λ . If the emitted wave is a sound, the received sound is deeper than the emitted sound. If the wave is light, the observed light is redder than the emitted light.

- Conversely, if the source moves closer to the listener, the second peak covers a smaller distance to reach the observer, a distance equal to $\lambda - d$. The observer receives a wave of wavelength

$$\lambda' = \lambda - d = Pc - v_r P = Pc \left(1 - \frac{v_r}{c} \right). \text{ The received wavelength is } \lambda \left(1 - \frac{v_r}{c} \right). \text{ Therefore, it is smaller than } \lambda.$$

If the emitted light is a sound, the received sound is more high-pitched than the emitted sound. If the wave is light, the observed light is more blue than the emitted light.

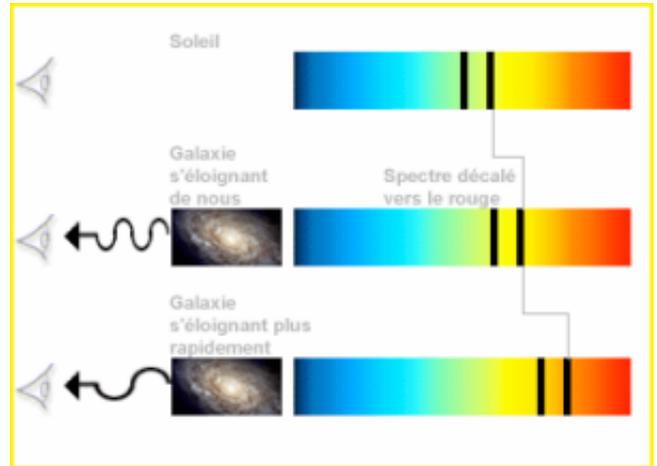
Applications of the Doppler effect

A motorbike, which passes near a listener, is a sound transmitter which moves closer to and away from the observer. The sound is more high-pitched during the approach, and deeper when the motorbike moves away.



The dark bands visible in the spectrum of the Sun are also seen in the spectra of galaxies (same physical process), but shifted to the red. In the real data, the arrows indicate the position of the dark bands in three more and more distant galaxies.

The very distant galaxies move away from us at high velocity. Their spectra are shifted very much to the red.



Doppler effect

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